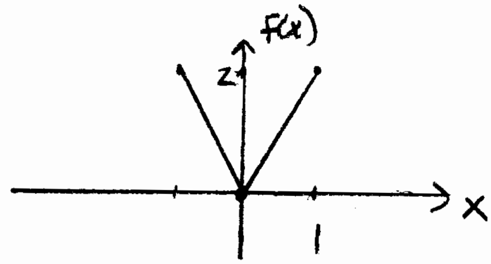
Solutions to Sample Test #3

#1) $f(x) = \begin{cases} -2x & -1 \le x \le 0 \\ 2x & 0 \le x \le 1 \end{cases}$



$L = 1$

since function is even, there will be no sin terms

$$a_0 = \frac{1}{2L}\int_{-1}^{1} f(x)\,dx = \frac{1}{2}\left[\int_{-1}^{0} -2x\,dx + \int_0^1 2x\,dx\right] = \underline{\underline{1}}$$

$$a_n = \frac{1}{L}\int_{-1}^{1} f(x)\cos\left(\frac{n\pi x}{L}\right)dx\Big]$$

$$= \int_{-1}^{0}(-2x)\cos(n\pi x)\,dx + \int_0^1 (2x)\cos(n\pi x)\,dx$$

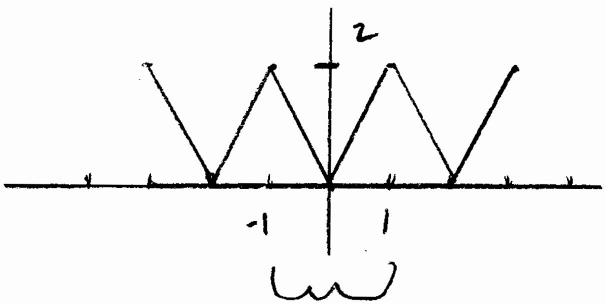
$$= \left(\frac{2\cos(n\pi)}{n^2\pi^2} + \cancel{\frac{2\sin(n\pi)}{n\pi}}^{\,0} - \frac{2}{n^2\pi^2}\right)$$

$$+ \left(\frac{2\cos(n\pi)}{n^2\pi^2} + \cancel{\frac{2\sin(n\pi)}{n\pi}}^{\,0} - \frac{2}{n^2\pi^2}\right)$$

$$\Rightarrow a_n = \frac{4}{n^2\pi^2}\left((-1)^2 - 1\right) = \begin{cases} 0 & n \text{ even} \\ \frac{-8}{n^2\pi^2} & n \text{ odd} \end{cases}$$

$$\boxed{f(x) \sim 1 - \frac{8}{\pi^2}\cos(\pi x) - \frac{8}{9\pi^2}\cos(3\pi x)}$$

① Continued: Plot of Series



Periodicity of 2

② complex Fourier series of same function

$$d_0 = a_0 = 1$$

$$d_n = \frac{1}{2L}\int_{-\infty}^{\infty} f(x)\, e^{-i\omega_n x}\,dx \Rightarrow \omega_n = \frac{n\pi}{L} = n\pi \qquad L=1$$

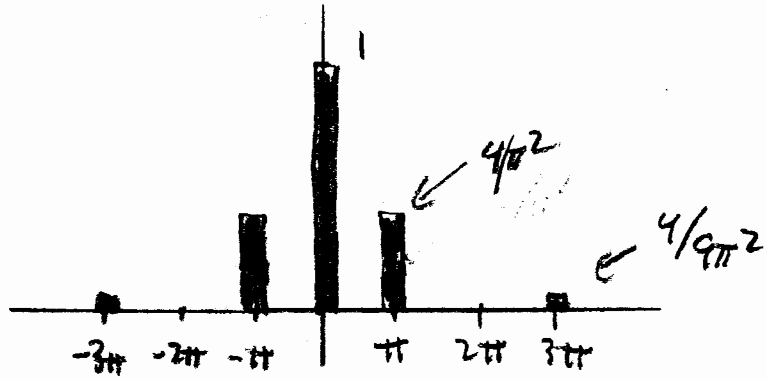
$$= \frac{1}{2}\int_{-1}^{0}(-2x)\, e^{-in\pi x}\,dx + \frac{1}{2}\int_{0}^{1} 2x\, e^{-in\pi x}\,dx$$

$$= \frac{(-1)^n-1}{n^2\pi^2} - \cancel{\left(\frac{(-1)^n}{n\pi}\right)} i + \left(\frac{(-1)^n-1}{n^2\pi^2}\right) + \cancel{\left(\frac{(-1)^n}{n\pi}\right)} i$$

$$= \frac{2}{n^2\pi^2}\left((-1)^n-1\right) \Rightarrow d_n = \begin{cases} 0 & n \text{ even} \\ \frac{-4}{n^2\pi^2} & n \text{ odd} \end{cases}$$

$$\boxed{\Rightarrow f(x) \sim \frac{-4}{9\pi^2}e^{-3\pi i x} - \frac{4}{\pi^2}e^{-\pi i x} + 1 - \frac{4}{\pi^2}e^{\pi i x} - \frac{4}{9\pi^2}e^{3\pi i x}}$$

2 continued: Frequency Spectrum for 5 terms



3

$$-\frac{4}{\pi^2}e^{-i\pi x} - \frac{4}{\pi^2}e^{+i\pi x}$$

$$= \frac{-4}{\pi^2}\left(\cos(\pi x) \cancel{- i\sin(\pi x)} + \cos(\pi x) \cancel{+ i\sin(\pi x)}\right)$$

$$= \frac{-8}{\pi^2}\cos(\pi x)$$

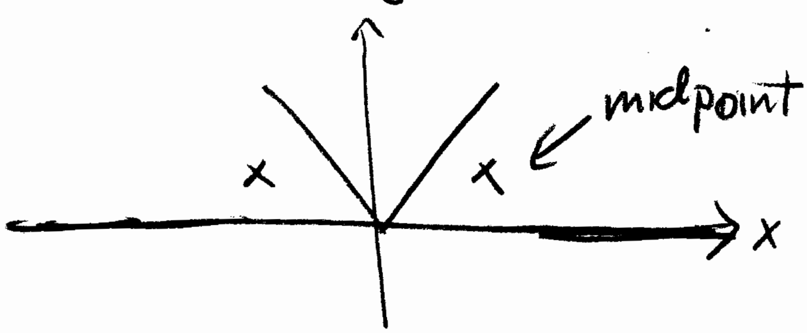
(4)

$$C_\omega = \int_{-1}^{1} f(x) e^{-i\omega x}\,dx$$

$$= \int_{-1}^{0} -2x\, e^{-i\omega x}\,dx + \int_{0}^{1} 2x e^{-i\omega x}\,dx$$

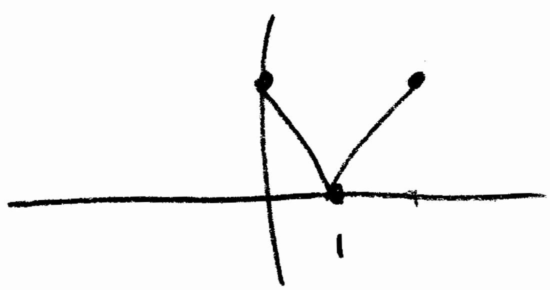
$$= \boxed{\frac{4}{\omega^2}\cos(\omega) + \frac{4}{\omega}\sin(\omega) - \frac{4}{\omega^2}}$$

$\Rightarrow$



(5) Sketch function

This is just the function from #1 but shift right 1



$\therefore$ use time shift and result in #4

$$\boxed{C_\omega = e^{-i\omega}\left(\frac{4}{\omega^2}\cos(\omega) + \frac{4}{\omega}\sin(\omega) - \frac{4}{\omega^2}\right)}$$