

Score:

Name:

Solutions

Section (circle one): 1 2 3 4 5 6

Team (circle one): a b c d e f

SM316 – Test #2– Fall 2006

Calculators/one note sheet/book allowed. Box/circle your final answer.

(Use no more than 2 decimals to answer all questions)

YOU MUST SHOW ALL WORK FOR FULL CREDIT.

1. (15 pts) A random sample of 5 students from this course had the following scores on the first SM316 Exam:

69 80 76 72.5 84

Calculate the mean and standard deviation of this sample.

$$\bar{x} = \frac{1}{5} (69 + 80 + 76 + 72.5 + 84) = \boxed{76.3}$$

$$s^2 = \frac{1}{4} [(69 - 76.3)^2 + (80 - 76.3)^2 + (76 - 76.3)^2 + (72.5 - 76.3)^2 + (84 - 76.3)^2] =$$
$$= \frac{1}{4} [53.29 + 13.69 + .09 + 14.44 + 59.29] = 35.2$$

$$\Rightarrow \boxed{s = 5.93}$$

2. (15 pts) Determine a 95% confidence interval for population mean for the sample in Problem 1 (Assume that the answer to Problem 1 was $\bar{x} = 76.3$ and $s = 5.93$).

$$t_{.025} \approx 2.78 \left\{ \begin{array}{l} \text{for 4} \\ \text{degrees} \\ \text{of freedom} \end{array} \right. t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$\Rightarrow \mu = \bar{x} \pm \frac{s}{\sqrt{n}} t_{.025}$$
$$= 76.3 \pm \frac{5.93}{\sqrt{5}} (2.78) = 76.3 \pm 7.37$$

$$\Rightarrow \boxed{68.93 \leq \mu \leq 83.67}$$

No marks on this table

1	
2	
3	
4	
5	
6	
7	
8	
cumm.	

3. (15 pts) Determine a 95% confidence interval for the population standard deviation in Problem 1 (Assume that the answer to Problem 1 was $\bar{x} = 76.3$ and $s = 5.93$).

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \Rightarrow \sigma^2 = \frac{(n-1)s^2}{\chi^2}$$

$$\chi^2_{0.025} = 11.143$$

$$\sigma^2 = \frac{(4)(5.93)^2}{11.143}$$

$$\Rightarrow \sigma^2 = 12.62$$

$$\Rightarrow \sigma = 3.55$$

$$\Rightarrow \boxed{3.55 \leq \sigma \leq 17.05}$$

$$\chi^2_{0.975} = 0.484$$

$$\sigma^2 = \frac{(4)(5.93)^2}{0.484}$$

$$\Rightarrow \sigma^2 = 290.62$$

$$\Rightarrow \sigma = 17.05$$

(for 4 degrees of freedom)

4. (15 pts) Find the probability that a sample in Problem 1 will have a standard deviation between 5.11 and 9.24 (Assume the answer to Problem 1 was $\bar{x} = 76.3$ and $s = 5.93$).

$$\chi^2_1 = \frac{(n-1)(s^2)}{\sigma^2} = \frac{(4)(5.93)^2}{(5.11)^2} = 5.39$$

$$\chi^2_2 = \frac{(4)(5.93)^2}{(9.24)^2} = 1.65$$

$$P(\chi \geq 5.39) = 0.25$$

$$P(\chi \geq 1.65) = 0.80$$

$$\Rightarrow P(1.65 \leq \chi \leq 5.39) = \boxed{0.55}$$

Name: _____

5. (15 pts) The amount of time that a student spends on each problem of an exam is $\bar{x} = 5$ minutes with a standard deviation of $\sigma = 2$ minutes. Assuming that this exam has eight questions, what is the probability that any given student will not finish the exam in 50 minutes?

$$\Rightarrow \mu = \frac{50}{8} = 6.25$$

$$\Rightarrow \sigma \text{ is known} \Rightarrow z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \quad \leftarrow \quad \underline{\underline{n=8}}$$

$$\Rightarrow z = \frac{5 - 6.25}{2/\sqrt{8}} = \underline{\underline{-1.78}}$$

$$P(z < -1.78) \approx \boxed{.038}$$

6. (10 pts) Construct a 98% confidence interval for the time required to complete the exam in Problem 4 (Hint: Find the confidence interval for μ)?

$$CI = \bar{x} \pm z_{.99} \frac{\sigma}{\sqrt{n}} \quad \Rightarrow \quad z_{.99} \approx 2.33$$

$$\Rightarrow \mu = 5 \pm (2.33) \left(\frac{2}{\sqrt{8}} \right) = 5 \pm 1.65$$

$$\Rightarrow \boxed{3.35 \leq \mu \leq 6.65}$$

min

7. (10 pts) In Problem 6, how many questions must be administered to obtain a 98% confidence interval for $\mu = 5 \pm 1$?

$$e = Z_{.99} \frac{\sigma}{\sqrt{n}} \Rightarrow 1 = 2.33 \left(\frac{2}{\sqrt{n}} \right)$$

$$\Rightarrow n = \left(\frac{(2.33)(2)}{1} \right)^2 = \underline{\underline{21.7}}$$

\therefore 22 Questions

8. (5 pts) In Problem 1, it is later determined that the population mean is 74. Obtain a 98% confidence interval for μ .

\Rightarrow a population mean is an exact entity. It has no confidence interval. Confidence Intervals are constructed about sample means