Sample Test 2 Solutions

#1) Variance is known

$\therefore \quad z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$

to complete 25 hands in 60 minutes requires $\mu = 60/25 = 2.4$

$$\Rightarrow z = \frac{2 - 2.4}{1/\sqrt{25}} = -2$$

$$\Rightarrow P(\mu \le 2.4) \Rightarrow P(z \ge -2) \simeq \underline{\underline{.98}}$$

25 hands per hour

#2) $\mu = \bar{x} \pm z_{.975}\left(\dfrac{\sigma}{\sqrt{n}}\right) = 2 \pm 1.96\left(\dfrac{1}{\sqrt{25}}\right)$

$= 2 \pm .39 \Rightarrow \boxed{1.61 \le \mu \le 2.39}$

#3) $e = z_{.99}\left(\dfrac{\sigma}{\sqrt{n}}\right) \Rightarrow .5 = 2.33\left(\dfrac{1}{\sqrt{n}}\right)$

$\Rightarrow \sqrt{n} = 2.33/.5 = 4.66 \Rightarrow n = 21.7$

$\Rightarrow \boxed{22 \text{ hands}}$

④ $\overline{X} = \frac{1}{9}(4+2+5+6+6+6+6+3+1) = \boxed{4.33}$

$$s^2 = \frac{1}{8}\sum_{n=1}^{9}(x_i - \overline{x})^2 =$$

$$= \frac{1}{8}\left(\left(-\tfrac{1}{3}\right)^2 + \left(-\tfrac{7}{3}\right)^2 + \left(\tfrac{2}{3}\right)^2 + \left(\tfrac{5}{3}\right)^2 + \left(\tfrac{5}{3}\right)^2 + \left(\tfrac{5}{3}\right)^2 - \left(\tfrac{4}{3}\right)^2 - \left(\tfrac{10}{3}\right)^2\right)$$

$$= \frac{1}{8}(30) = 3.75$$

$$\Rightarrow \boxed{s = 1.94}$$

⑤ σ is unkown $\Rightarrow$ use t-distrubution

$$\Rightarrow t = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} \Rightarrow \mu = \overline{x} \pm t_{.1}\frac{\sigma}{\sqrt{n}}$$

for 80% C.I
(use 8 degrees of freedom)

$$\Rightarrow \mu = 4.33 \pm 1.397\left(\frac{1.94}{\sqrt{9}}\right)$$

$$\Rightarrow \mu = 4.33 \pm .90$$

$$\Rightarrow \boxed{3.43 \le \mu \le 5.23}$$

(6) $\chi^2 = (n-1)\frac{s^2}{\sigma^2} \Rightarrow \sigma^2 = \frac{8(1.94)^2}{\chi^2}$

8 degrees of freedom
$$\begin{cases} \chi^2_{.05} = 15.507 \Rightarrow \sigma^2 = 1.94 \Rightarrow \sigma = 1.39 \\ \chi^2_{.95} = 2.733 \Rightarrow \sigma^2 = 11.02 \Rightarrow \sigma = 3.32 \end{cases}$$

$\Rightarrow \boxed{1.39 \leq \sigma \leq 3.32}$

(7) $\Rightarrow 1.5 \leq \sigma \leq 2.94 \Rightarrow 2.25 \leq \sigma^2 \leq 8.64$

$\chi^2_1 = (8)\left(\frac{(1.94)^2}{2.25}\right) = 13.39$

$\chi^2_2 = (8)\left(\frac{(1.94)^2}{8.64}\right) = 3.48$

$P(\chi^2 \geq 13.38) \approx .1$ $\quad$ $P(\chi^2 \geq 3.48) \approx .9$

$\Rightarrow P(3.48 \leq \chi^2 \leq 13.38) \approx .8$

8) now σ is known

$$\therefore \quad Z = \frac{4.33 - 3.5}{(1.44)/\sqrt{9}} = 1.73$$

$$\Rightarrow P(\mu \geq 4.33) = Pr(Z \geq 1.73)$$

$$= 1 - .9582 \approx \underline{\underline{.04}}$$