

Score:

Name: _____

Section (circle one): 1 2 3 4 5 6

Team (circle one): a b c d e f

SM316 – Test #1 – Fall 2006

Closed book. Calculators/one note sheet allowed. Properly label all graphs. Box/circle your final answer. **YOU MUST SHOW ALL WORK FOR FULL CREDIT.**

1. (10 pts) Consider the following experiment consisting of 2 events: Roll a six sided die. If 1 or 2 shows up then roll the die one more time. If 3 shows up draw from a bag of balls that contains four possible colors (red/yellow/blue/green). If 4, 5, or 6 shows up flip a two sided coin (heads/tails). What is the sample space for this experiment?

$$S = \{ 1H, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, \\ 3R, 3Y, 3B, 3G, 4H, 4T, 5H, 5T, 6H, 6T \}$$

2. (10 pts) Determine the following probabilities:

a. P(red ball):

$$P\{3R\} = \left(\frac{1}{6}\right) \left(\frac{1}{4}\right) = \underline{\underline{\frac{1}{24}}}$$

b. P(red ball|4):

$$P(R|4) = \frac{P(R \cap 4)}{P(4)} = \underline{\underline{0}}$$

c. P(heads):

$$P(4H \cap 5H \cap 6H) = \\ \left(\frac{1}{6}\right) \left(\frac{1}{2}\right) + \left(\frac{1}{6}\right) \left(\frac{1}{2}\right) + \left(\frac{1}{6}\right) \left(\frac{1}{2}\right) = \underline{\underline{\frac{1}{4}}}$$

No marks on this table

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3. (10 pts) A random sample of 200 midshipmen is classified as follows:

Major	Male	Female	
Mathematics	38	45	83
History	28	50	78
Engineering	22	17	39
	88	112	200

If a person is picked at random from this group, find:

a. $P(\text{male}|\text{history}) = \frac{28}{28+50} = \frac{14}{39}$

b. $P(\text{history}|\text{male}) = \frac{28}{38+28+22} = \frac{7}{22}$

c. $P(\text{male and history})$.

$$\frac{28}{200} = 0.14$$

d. $P(\text{male or history})$.

$$P(\text{male}) + P(\text{history}) - P(\text{male} \cap \text{history})$$

$$= \frac{88}{200} + \frac{78}{200} - \frac{28}{200} = 0.69$$

4. (10 points) The probability that the captain of a ship is in his stateroom when the new Ensign reports aboard is 0.35. Given that the Captain is in his stateroom, the probability that has time to talk to the new Ensign is .7. Find the probability that the Captain is in his stateroom and has time to talk to the new Ensign.

A = CO in stateroom

B = CO wants to talk to Ensign

$$P(A) = 0.35 \Rightarrow P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(B|A) = 0.7$$

$$\Rightarrow 0.7 = \frac{P(A \cap B)}{0.35} \Rightarrow P(A \cap B) = 0.245$$

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5. (10 points) Let W be a ^{three} random variable giving twice the number of heads minus thrice the number of tails in ~~two~~ coin tosses. List the elements of the sample space S for the three tosses. What is probability distribution of the random variable W if the coin is biased so that tails occurs three times as often as heads?

$$S = \{ HHH, HHT, HTH, HTT, THH, THT, TTH, TTT \}$$

$$= \{ 6, 1, 1, -4, 1, -4, -4, -9 \}$$

$$P(H) = \frac{1}{4} \quad P(T) = \frac{3}{4}$$

$$P(6) = \left(\frac{1}{4}\right)^3 = \frac{1}{64}$$

$$P(1) = 3\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right) = \frac{9}{64}$$

$$P(-4) = 3\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^2 = \frac{27}{64}$$

$$P(-9) = \left(\frac{3}{4}\right)^3 = \frac{27}{64}$$

X	P(x)
6	1/64
1	9/64
-4	27/64
-9	27/64

6. (10 points) An unfortunate plebe loses a bet and is now selling lemonade on Stribling Walk to raise money for his company's dining out. Suppose he purchases 5 gallons of lemonade for \$1.50 per gallon and is able to resale this at \$4.50 per gallon. If the probability distribution of the random variable X , the number of gallons sold is:

x	0	1	2	3	4	5
f(x)	1/10	1/5	1/5	2/5	1/15	1/30

What is the expected profit?

$$P = 3(x) - 1.5(5-x)$$

$$= 3x - 7.5 + 1.5x = 4.5x - 7.5$$

$$E(4.5x - 7.5) = 4.5E(x) - 7.5$$

$$E(x) = \frac{0}{10} + \frac{1}{5} + \frac{2}{5} + \frac{6}{5} + \frac{4}{15} + \frac{5}{30} = 2.2\bar{3}$$

$$\Rightarrow E(P) = 4.5(2.2\bar{3}) - 7.5 = \underline{\underline{\$2.55}}$$

7. (10 points) Research suggests that 38% of the Class of 2007 decided to apply to the Navy Academy after watching "Top Gun". Find the probability that among 7 members of 2007:
- Exactly 5 were motivated to apply after watching "Top Gun".

$$\binom{7}{5} (.38)^5 (.62)^2 = \underline{\underline{.0634}}$$

- At most 5 were motivated to apply after watching "Top Gun".

$$P(X=6) = \binom{7}{6} (.38)^6 (.62) = .0131$$

$$P(X=7) = \binom{7}{7} (.38)^7 = .0011$$

$$P(X \geq 6) = .0142$$

$$\Rightarrow \underline{\underline{P(X \leq 5) = .9858}}$$

- What is the mean and variance of the above distribution?

$$\mu = np = 7(.38) = \underline{\underline{2.66}}$$

$$\sigma^2 = npq = (2.66)(.62) = \underline{\underline{1.65}}$$

8. (10 points) The total of Mountain Dew in gallons consumed by midshipmen on a daily basis is a random variable X having a continuous uniform distribution with $100 \leq X \leq 250$. What is the probability that Mountain Dew consumption by Midshipmen is:
- Exactly 125 gallons?

0

- Between 113 and 132 gallons?

$$P(113 \leq X \leq 132) = \frac{132-113}{250-100} = \frac{19}{150} = \underline{\underline{.1267}}$$

- What is the average amount of Mountain Dew consumed?

$$\underline{\underline{\mu}} = \frac{250+100}{2} = \underline{\underline{175 \text{ gal}}}$$

- What is the standard deviation of the amount of Mountain Dew Consumed?

$$\sigma^2 = \frac{(250-100)^2}{12} = 1875$$

$$\underline{\underline{\sigma}} = \underline{\underline{43.3 \text{ gal}}}$$

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9. (15points) Given the following results on this test: $\mu = 82.3$ and $\sigma = 11.2$ with the following grading criteria:

A	$X > 89.5$
B	$79.5 < X < 89.5$
C	$69.5 < X < 79.5$
D	$59.5 < X < 69.5$
F	$X < 59.5$

Assuming a normal distribution, find the following probabilities:

- e. A student will fail the test.

$$P(X < 59.5) = P\left(Z < \frac{59.5 - 82.3}{11.2}\right) = -2.036$$

$$P(Z < -2.036) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-2.036} e^{-\frac{1}{2}x^2} dx = \underline{\underline{.0209}}$$

- f. A student will score a 75.

$$P(X = 75) = 0$$

- g. A student will get a C or higher on the test.

$$P(X \geq 69.5) = P(Z \geq -1.14) = \frac{1}{\sqrt{2\pi}} \int_{-1.14}^{\infty} e^{-\frac{1}{2}x^2} dx$$
$$\approx \underline{\underline{.8729}}$$

- h. A student will get an D, C, or B.

$$P(59.5 < X < 89.5)$$
$$= P(-2.036 < Z < .6429) = \frac{1}{\sqrt{2\pi}} \int_{-2.036}^{.6429} e^{-\frac{1}{2}x^2} dx$$
$$\approx \underline{\underline{.719}}$$

10. (5 points - 4.0 buster) Show that $\sigma_{ax^2+b}^2 = a^2 \sigma_{x^2}^2$.

$$\begin{aligned}\sigma_{ax^2+b}^2 &= E([ax^2+b]^2) - [E(ax^2+b)]^2 \\ &= E(a^2x^4 + 2abx^2 + b^2) - (aE(x^2) + b)^2 \\ &= a^2 E(x^4) + 2abE(x^2) + b^2 - a^2(E(x^2))^2 - 2abE(x^2) - b^2 \\ &= a^2(E(x^4) - E(x^2)^2) \\ &= \boxed{a^2 \sigma_{x^2}^2}\end{aligned}$$