Score:

Name: Solutions
Section (circle one): 1 2 3 4 5 6
Team (circle one): a b c d e f

SM316 – Quiz #9 (Section 20.1, 14.7) – Due Monday

Take home quiz, open book, open notes. You may work with team members to solve problems, but you may not copy another's work. Calculators are allowed, but you must show all work for full credit.

1. Find the solution to: $\left|\left(\frac{(1+i)^4(-1-i)}{2-i}\right)^{\frac{1}{2}}\right|$. Express answer to 2 decimal places. Hint: $|re^{i\theta}| = r$

polar form

First do $(1+i)^4 = \left(\sqrt{2}e^{i\frac{\pi}{4}}\right)^4 = 4e^{i\pi} = -4$

Then: $-4(-1-i) = 4+4i$

Then $\frac{4+4i}{2-i}\frac{(2+i)}{(2+i)} = \frac{8+8i+4+4i^2}{2^2+1^2} = \frac{4+12i}{5}$ (−4)

$= \frac{4}{5} + \frac{12}{5}i \Rightarrow$ convert to polar

$\Rightarrow r = \left(\left(\frac{4}{5}\right)^2 + \left(\frac{12}{5}\right)^2\right)^{1/2} = \sqrt{\frac{160}{25}} = \frac{4\sqrt{10}}{5}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{12/5}{4/5}\right) = \tan^{-1} 3 \approx 1.25$

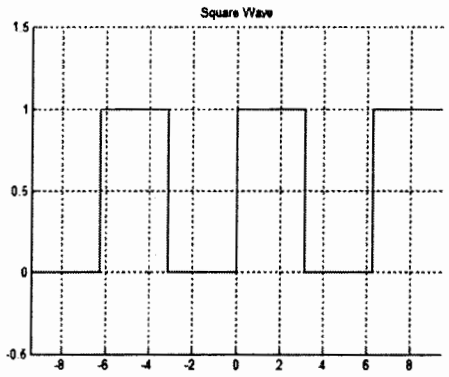
$\Rightarrow \left(\frac{4}{5} + \frac{12}{5}i\right)^{1/2} \approx \left(\frac{4\sqrt{10}}{5}e^{1.25i}\right)^{1/2}$

take square root

$\sqrt{\frac{4\sqrt{10}}{5}}e^{\frac{1.25}{2}i} \approx 1.59e^{.63i}$

Finally $\left|1.59e^{.63i}\right| \approx 1.59$

2. Determine the first five terms of the complex Fourier series ($-2 \le n \le 2$) for the square-wave depicted on the right where the following function repeats with 2π - periodicity:



$$f(x) = \begin{cases} 0 & -\pi \le x \le 0 \\ 1 & 0 \le x \le \pi \end{cases} \Rightarrow \boxed{T = 2\pi}$$

Plot the frequency spectrum for the five terms in the space provided.

$$c_0 = \frac{1}{2\pi}\int_{-\pi}^{0} 0\,dx + \frac{1}{2\pi}\int_{0}^{\pi} dx = \frac{1}{2\pi}(\pi - 0) = \boxed{\frac{1}{2}}$$

$$c_n = \frac{1}{2\pi}\int_{0}^{\pi} e^{-\frac{in\pi x}{\pi}}\,dx = \frac{1}{2\pi}\int_{0}^{\pi} e^{-inx}\,dx$$

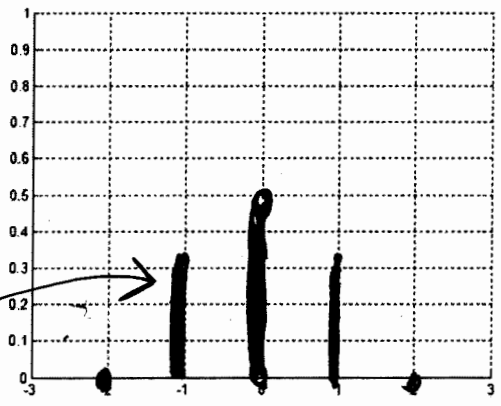
$$= \frac{\sin(n\pi)}{2n\pi} + \frac{\cos(n\pi) - 1}{2n\pi}\,i = \frac{(-1)^n - 1}{2n\pi}\,i$$

$$= \begin{cases} 0 & n \text{ even} \\ \frac{-i}{n\pi} & n \text{ odd} \end{cases} \Rightarrow \boxed{f(x) \sim \frac{i}{2\pi}\sum_{-\infty}^{\infty} \frac{(-1)^n - 1}{n} e^{inx}}$$

$\underline{n=-2}$ $\underline{n=-1}$ $\underline{n=0}$ $\underline{n=1}$ $\underline{n=2}$

$$0 \quad + \frac{-i}{-1\pi}e^{-ix} + \frac{1}{2} + \frac{-i}{\pi}e^{ix} + 0$$

$$\boxed{\cdots \frac{i}{\pi}e^{-ix} + \frac{1}{2} - \frac{i}{\pi}e^{ix}}$$



$\approx .318$