Score:

Name:
Section (circle one): 1 2 3 4 5 6
Team (circle one): a b c d e f

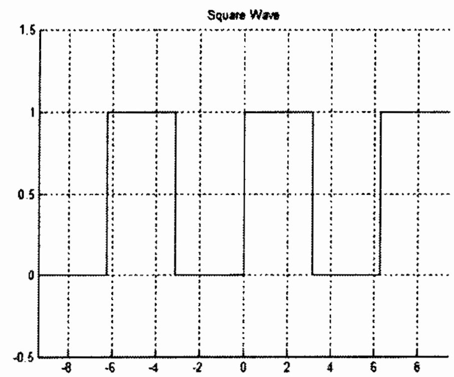
SM316 – Quiz #8 (Section 9.4) – Due Monday

Take home quiz, open book, open notes. You may work with team members to solve problems, but you may not copy another's work. Calculators are allowed, but you must show all work for full credit.

1. Determine the first five non-zero terms for the square-wave depicted on the right where the following function repeats with 2π -periodicity:

$$f(x) = \begin{cases} 0 & -\pi \le x \le 0 \\ 1 & 0 \le x \le \pi \end{cases}$$

Plot the series as the number of terms approaches infinite in the space provided.



$$L = \pi \Rightarrow a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{2\pi}\int_0^{\pi} dx = \boxed{\frac{1}{2}}$$

$$a_n = \frac{1}{\pi}\int_0^{\pi}(1)\cos\left(\frac{n\pi x}{\pi}\right)dx = \frac{1}{\pi}\int_0^{1}\cos(nx)\,dx$$

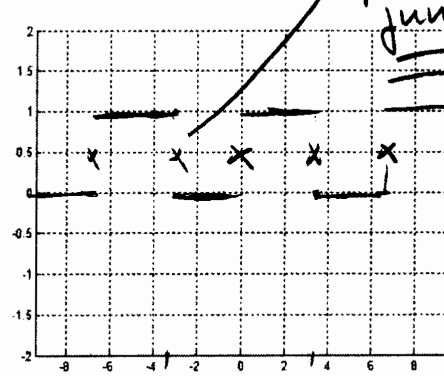
$$= \frac{1}{n\pi}\sin(nx)\Big|_0^{\pi} = \frac{1}{n\pi}\sin(n\pi) = 0 \Rightarrow \boxed{a_n = 0} \text{ for } n > 0$$

$$b_n = \frac{1}{\pi}\int_0^{\pi}\sin(nx)\,dx = \frac{1}{n\pi}\left(-\cos(nx)\Big|_0^{\pi}\right)$$

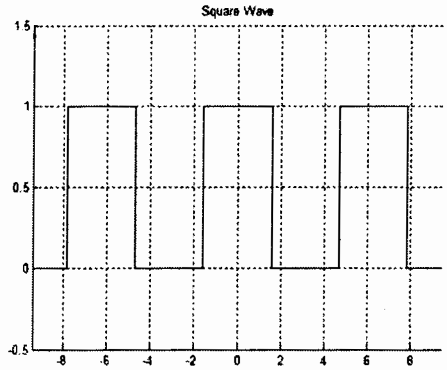
$$= \frac{1}{n\pi}\left(-\cos(n\pi) + \cos(0)\right) = \frac{1}{n\pi}\left(1-(-1)^n\right) = \begin{cases} 0 & n = \text{even} \\ \frac{2}{n\pi} & n = \text{odd} \end{cases}$$

$$\boxed{f(x) \sim \frac{1}{2} + \frac{2}{\pi}\sin(x) + \frac{2}{3\pi}\sin(3x) + \frac{2}{5\pi}\sin(5x) + \frac{2}{7\pi}\sin(7x)}$$

central point at jump!



2. Determine the first five non-zero terms for the square-wave depicted on the right where the following function repeats with 2π -periodicity:



$$f(x)=\begin{cases}0 & -\pi \le x \le -\pi/2 \\ 1 & -\pi/2 \le x \le \pi/2 \\ 0 & \pi/2 \le x \le \pi\end{cases}$$

One of the main terms in the series goes to zero. Why

$$a_0 = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} dx \Rightarrow \boxed{a_0 = \frac{1}{2}}$$

$$a_n = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} \cos(nx)\,dx = \frac{1}{n\pi}\sin(nx)\Big|_{-\pi/2}^{\pi/2} = \frac{1}{n\pi}\left(\sin\left(\frac{n\pi}{2}\right) - \sin\left(\frac{-n\pi}{2}\right)\right)$$

$$= \frac{1}{n\pi}\left(\sin\left(\frac{n\pi}{2}\right) - \left(-\sin\left(\frac{n\pi}{2}\right)\right)\right) = \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right) = \begin{cases}0 & n = \text{even} \\ \frac{2}{n\pi} & n = 1, 5, 9\ldots \\ -\frac{2}{n\pi} & n = 3, 7, 11\ldots\end{cases}$$

$$b_n = \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} \sin(nx)\,dx = \frac{1}{n\pi}\cos(nx)\Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{n\pi}\left(\cos\left(\frac{n\pi}{2}\right) - \cos\left(-\frac{n\pi}{2}\right)\right) = \frac{1}{n\pi}\left(\cos\left(\frac{n\pi}{2}\right) - \cos\left(\frac{n\pi}{2}\right)\right) = \underline{0}$$

$\uparrow$ Even function

$$\therefore \boxed{f(x) = \frac{1}{2} + \frac{2}{\pi}\cos(x) - \frac{2}{3\pi}\cos(3x) + \frac{2}{5\pi}\cos(5x) - \frac{2}{7\pi}\cos(7x)}$$

$\Rightarrow$ since the original function is <u>even</u>, the "sin" terms fall out because $\sin\frac{n\pi x}{L}$ is <u>odd</u>